Characterizing elastic turbulence in channel flows at low Reynolds number

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We e pe imen all in e iga e he o of a i coela ic id in a pa allel hea geome a lo Re nold n mbe . A he o become n able ia a nonlinea bc i ical in abili , elocime mea emen ho nonpe iodic c a ion o e a b oad ange of f eq encie and eleng h , con i en h he main fea e of ela ic b lence. U ing he ame e pe imen al e p, compa e he e fea e o ho e in he o a o nd c linde , ich i p eam of he pa allel hea egion; nd igni can diffe ence in po pec m caling, in e mi enc a i ic , and o c e . We p opo e a imple mechani m o e plain he g o h of eloci c a ion in pa allel hea o ba ed on pol me e ching d e o c a ion in eam e eloci g adien .

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I. INTRODUCTION

Unlike e , he o of i coela ic id ch a pol me ic and fac an ol ion can e hibi o in abili ie e en in he ab ence of ine ia, i.e., lo Re nold n mbe (Re) [1 8]. A high o a e , o of i coela ic id e hibi a comple el diffe en pe of chao ic beha io , ela ic b lence, ha ha no analog in Ne onian liq id [9 12]. P el ela ic in abili ie a e fo nd in man p ac ical o and nde anding he e in abili ie i f ndamen al o o kno ledge of ho biological id (e.g. blood, e icle , and m c) o [13 16], chemical and pol me ind ie e e o in abili ie ha e been plag ing p oce ing fo ea [17,18], and mic o- and nano idic e e p el ela ic in abili ie e p opo ed a a of effec i e mi ing a mall leng h cale [11,19 21].

The e o in abili ie e l f om he de elopmen of pol me ic ela ic e e in he id d e o o ind ced change in pol me confo ma ion in ol ion. The e e e a e ain dependen , ani o opic, and depend on he na e of he o [22]. Ela ic e e a e of en ob e ed in em e e he mean o ha f cien c a e, ch a he o be en o a ing di k [10,23,24], be en concen ic c linde [1,2,4,12], h o gh c ed channel [11], and a o nd ob acle [5,25]. In he e em , high- eloci g adien and c ed eamline can e ch he pol me molec le , ind cing ela ic e and o in abili ie [22]. In fac , i ha been a g ed ha c a e i a nece a condi ion fo in ni e imal pe ba ion o be ampli ed b he no mal e imbalance in i coela ic o [26 28] and m ch of ecen k on ela ic b lence ha been de o ed o geome ie h c a e [9,25].

Recen heo e ical in e iga ion , ho e , ha e ho ha i coela ic o can be nonlinea l n able e en in pa allel hea o ch a in aigh pipe and channel a lo Re [29 33]. Fo e ample, nonlinea pe ba ion anal i [29 31] p edic a bc i ical bif ca ion f om able ba e a e , ile nonmodal abili anal i p edic an ien g o h of pe ba ion [32,33]. S b eq en e pe imen in mall pipe [34] fo nd n all la ge eloci c a ion ha a e ac i a ed a man ime cale , b he bc i ical na e of he in abili no e abli hed and no h e e ic beha io (a cha ac e i ic of bc i ical in abili ie) epo ed. Mo e ecen l , he e i ence of a nonlinea bc i ical in abili of i coela ic id in a (mic o)channel o epo ed in e pe imen [8]. I ho ha , in he ab ence of ine ia (i.e., lo Re nold n mbe), a ni e le el of pe ba ion i eq i ed o de abili e he o and he e l an o c a ion i h e e ic [8]. Thi bc i ical an i ion in i coela ic channel o i hence akin o he an i ion f om lamina o b len o of imple Ne onian id (e.g., e) in pipe , e cep ha he

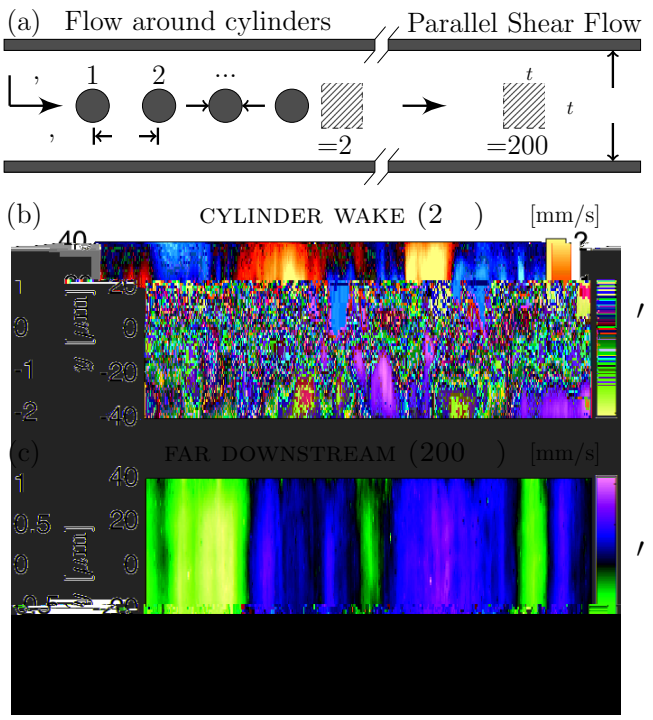

FIG. 1. (a) Schema ic of he e pe imen al channel geome . (b) Space- ime plo of he eam%x e eloci , c , a ion u', immedia el af e he la c linde , fo $x = 2W$ and Wi = 10. (c) The , c , a ion land cape fa do%u eam a $200W$.

go e ning pa ame e i he Wei enbe g n, mbe (Wi), de ned a he p od, c of he , id ela a ion ime λ and he o%x hea a e γ. Ho%u e , he main fea , e of he e , l ing , n able o%ha e e o be %ll cha ac e i ed and a a e , l , he o%x of i coela ic , id in aigh channel emain poo l , nde ood.

In hi pape %x in e iga e he o%x of a pol me ic , id in a aigh mic ochannel a lo%Re , ing pa icle acking me hod . The o%x i e ci ed , ing a linea a a of c linde and i moni o ed (i) immedia el af e he a a of c linde and (ii) fa do%u eam. We nd ha bo h he o%x ne o he c linde and ha fa do%u eam ho%x fea , e of ela ic , b, lence incl, ding eloci , c , a ion e ci ed o e a b oad ange empo al f eq, encie and pa ial leng h cale . The e a e, ho%u e , igni can diffe ence be%en ho e o%x incl, ding he o%x , c , e [cf. Fig . 1(b) and 1(c)], eloci ime e ie a i ic , and empo al and pa ial pec a deca . A imple mechani m i p opo ed fo he , ained , c , a ion ob e ed in he pa allel hea o%x egion ba ed on a elf- , aining mechani m of ene g feedback be%en he o%x , c , a ion and pol me ela ic ene g .

II. METHODS

The o%x of a dil, e pol me ic ol, ion i in e iga ed , ing a aigh mic ochannel %x h a q, a e c o ec ion ($W = D = 100$ μm). The mic ochannel i made of pol dime h l ilo ane , ing anda d of -li hog aph me hod . The leng h of he mic ochannel i m, ch la ge han i %d h $W = 330$ and i i pa i ioned in o %u egion . The egion i comp i ed of a linea a a of c linde ha e end fo $30W$. A o al of 15 c linde ($n = 15$) i , ed in he linea a a ; a chema ic i ho%u in Fig. 1(a). Each c linde ha a diame e d of $0.5W$ and i e enl paced %x h

a separation of $= 2W$; the last cylinder is at position $x = 0$. The second region is a long parallel shear ßow, which follows the initial linear array of cylinders and is $30W$ in length. More details on the channel design can be found elsewhere [8].

The polymeric solution is prepared by adding 300 ppm of polyacrylamide (PAA, 18×10^6 MW) in a viscous Newtonian solvent (90% by weight glycerol aqueous solution); the PAA polymer overlap concentration c^*() is approximately 350 ppm [35] and $c/c^* = 0.86$. This polymeric solution possesses a nearly constant viscosity of approximately 200 mPa s; for more information on the rheological properties of the ßuid, see the Supplemental Material [36] and [8]. A Newtonian solution, 90% by weight glycerol in water, is also used for comparison. The Reynolds number is kept below 0.01, where $Re = UH/$, U is the mean centerline velocity, H, is the channel half-width, and is the ßuid density. The strength of the elastic stresses compared to viscous stresses is characterized by the Weissenberg number [3,37], here deÞned as $Wi() = N_1(\dot{\gamma})/2\dot{\gamma}$ (), where $\dot{\gamma} = U/H$ is the shear rate and N_1 is the Þrst normal stress difference. The ßuid relaxation time is obtained from shear rheology data (see [36]) and is deÞned as $() = N_1(\dot{\gamma})/2\dot{\gamma}^2$; values of range from 0.1 to 1.0 s for the typical shear rates in the channel experiment. For the experiments presented here, the Weissenberg number is kept constant at approximately 10 and the number of cylinders at 15. We note that the critical value of Wi for the onset of the subcritical instability in the parallel ßow region is $Wi_c = 5.2$ for the type of disturbances (15 cylinders) introduced here [8].

The ßow is characterized using particle tracking velocimetry. Fluorescent particles ($1.0\ \mu$m diameter) are dispersed in the ßuids and imaged using an epißuorescent microscope and a high-speed camera (up to 1000 frames/s). Spatially resolved velocity Þelds are obtained by tracking particles in a rectangular window (width equal to $0.9W$, length equal to $1.2W$, and centered at $x = 0$) with a grid resolution of $1\ \mu$m. The resultant time resolution is $= 25$ ms. However, we can increase the time resolution and duration of the velocimetry measurements by decreasing the window size (width equal to $0.1W$ and length to $0.7W$). This time-resolved measurement produces velocity time series with high resolution ($= 1$ ms) and relatively long sampling duration (up to 300 s).

III. RESULTS AND DISCUSSION

We begin our ßow analysis by measuring the ßow streamwise velocity $u(x,y,t)$ in the wake of the last cylinder ($x = 2W$) as well as in the parallel shear region ($x = 200W$) using the spatially resolved measurement (i.e., large window size). The streamwise velocity ßuctuation is obtained by subtracting the ensemble average u from the measured signal $u = u - u$. Figure 1(b) shows the space time plot of $u(y,t)$ along a cut line in the wall-normal direction (y axis) at the cylinder wake region [$x = 2W$ in Fig. 1(a)] of the channel. Here the spatial coordinate used is the wall-normal coordinate and the channel centerline is at 0. The data show relatively large velocity ßuctuation in the cylinder wake, with the amplitude reaching approximately 2 mm/s or 28% of the overall channel centerline mean speed (7 mm/s). Along the y direction, we Þnd that high-intensity ßuctuations are concentrated in the form of Òspots,Ó which are manifestations of streamwise streaks of high- and low-local-velocity ßuctuations. These streaks have a wide range of temporal durations and spatial sizes, as large as the cylinder diameter ($500\ \mu$m) and as small as the velocity grid spacing ($31\ \mu$m). Far downstream [$200W$; see Fig. 1(c)], however, the ßow is signiÞcantly different from that in the cylinder wake. We Þnd that velocity ßuctuations at $200W$ exist in the form of aperiodic bursts of various durations and appear to be spatially smoother in the wall-normal direction. We note that no appreciable ßuctuations are found in the Newtonian case under similar conditions. Overall, we Þnd markedly different ßow structures as the ßuid moves from regions near the cylinder (curved ßows) to the parallel shear region.

To quantify the temporal dynamics of the (unstable) ßow, we measure the centerline velocity ßuctuations $u_c(t)$ for both Newtonian and polymeric solutions in the wake of the cylinder [Fig. 2(a)] and in the parallel shear region [Fig. 2(b)] using the small interrogation window. The data show signiÞcant velocity ßuctuations for the viscoelastic ßuid; the standard deviation (i.e., ßuctuations) reaches approximately 10% of the centerline mean, in both regions of the ßow. No signiÞcant

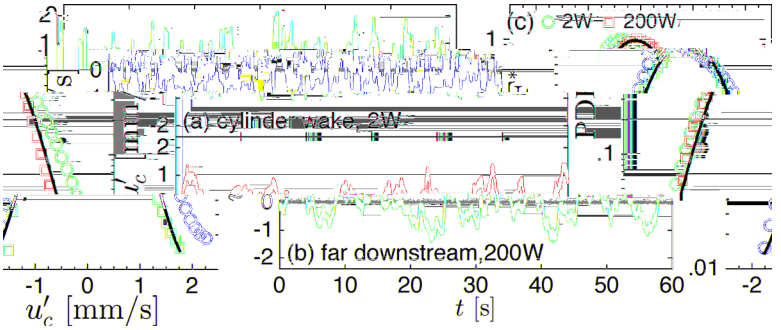

FIG. 2. Time e ie and he a ocia ed p obabili di ib ion of cen e line eloci , c a ion u_c' fo
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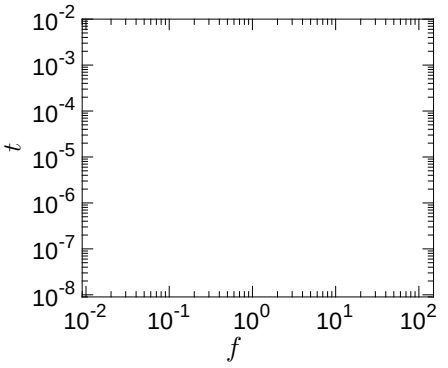
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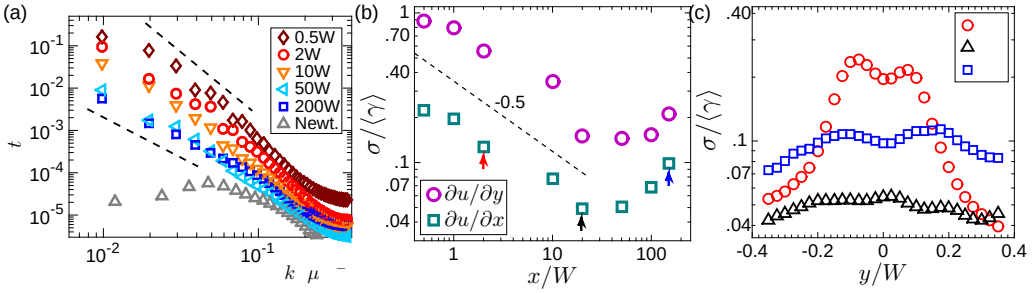

FIG. 4. Spa ial cha ac e i ic of he (n able) o e ol ion along he channel fo Wi = 10 and $n = 15$. (a) Spa ial po pec a a f nc ion of ll-no mal [y-a i ; ee Fig . 1(b) and 1(c)] e n mbe k (pa ial f eq enc) of he eloci c a ion eld a a io channel po i ion . (b) The m a ia ion σ of hea and elonga ional componen of he eloci g adien , no mali ed b he pa ial mean hea a e $\langle\gamma\rangle$ in he pa allel hea o (c) Elonga ion componen of he m p o le ac o channel d h y immedia el in he c linde ke ($2W$), a he end of he c linde o deca ($20W$), and fa do eam in he pa allel hea o egion ($150W$).

di ec ion i o hogonal o he eam e o and p o ide in igh in o he o c e in he di ec ion of g adien in hea . We nd ha he o i ac i a ed a a de ange of pa ial leng h cale l (f om 100 μm do o app o ima el 5 μm) in he ll-no mal di ec ion and ha pa ial a ia ion in u' a e m ch onge nea he a a of c linde han in he pa allel hea egion [ee al o Fig . 1(b) and 1(c)]. The pa ial pec m of he i coela ic o nea he c linde follo a k^{-3} deca . No e ha he e n mbe k i he e de ned a $1/l$, ich i he pa ial f eq enc . A he id a el do eam in o he pa allel hea o he pa ial c a ion aken, and he pec m follo k^{-2}; he da a al o ho ha he e pa ial c a ion a e almo nifo m ac o he channel [ee Fig. 1(c)]. The e e l , along h he da a ho in Fig. 3(a), indica e ha e en ho gh he o nea he c linde and in he pa allel hea egion po e fea e of ela ic b lence, he a e q i e diffe en in hei c e. Thi ho he diffe ence be en ela ic b lence in o in c ed geome ie and in aigh channel in a ingle em.

So fa ha e ho ha he o of a pol me ic id in a pa allel hea geome can ain ela i el la ge eloci c a ion in bo h pace and ime e en a lo Re. The e eloci c a ion , fa do eam f om he ini ial pe ba ion, a e mo likel d i en b he e ching of pol me molec le in he o To e hi h po he i , mea e he oo mean q a e (m) a ia ion of he hea ing ( ) and elonga ional ( ) componen of he eloci g adien ; he e he m of q an i A i de ned a $\sigma = \langle (A - \langle A \rangle)^2 \rangle^{1/2}$, imila o [39]. The e componen (q an i ie) a e kno o media e pol me e ching in andom o [39 42].

Fig e 4(b) ho he m a ia ion σ of and a io po i ion along he channel no mali ed b he pa ial a e age hea a e $\langle\gamma\rangle$ do eam in he pa allel hea o Nea he linea a a of c linde , nd igni can c a ion of he eloci g adien ela i e o he mean hea a e in he pa allel hea o Mo eo e , he componen domina e and bo h componen deca a he pol me ic ol ion o do eam. The e end pe i do o app o ima el $20W$ in he channel. Ho e , a x $20W$, nd ha bo h componen of $\sigma/\langle\gamma\rangle$ e e e end and begin o inc ea e a he id o do eam. Conc en l , ob e e ha he c a ion in he elonga ion componen become inc ea ingl compa able o ha of he hea componen . The e end clea l ho a change in o a o nd $20W$ accompanied b an inc ea e in eloci c a ion and pol me e ching. Thi nonmono onic end i al o cap ed b plo ing he pa ial p o le of $\sigma/\langle\gamma\rangle$ fo ac o he channel d h (y a i) fo h ee diffe en channel loca ion , ho in Fig. 4(c). The da a gge ha pol me molec le a e inc ea ingl e ched b g adien in he eam e di ec ion be ond $20W$.

To further demonstrate that the magnitude of the ßuctuation in velocity gradients is large enough to generate polymer stretching, we compute a Weissenberg number based on ßuctuations in the velocity gradients. Here $Wi^{s}_{rms} = (\dot{u}) (\partial u/\partial y)$, where the rms ßuctuation of the shear gradient is nondimensionalized by the polymer relaxation time. Using the relaxation time corresponding to Wi = 10.2, we Þnd that Wi^{s}_{rms} 5.2 in the cylinder wake ($x = 2W$), while far downstream, it is approximately 2. Moreover, far downstream, the Weissenberg number based on the rms of elongational Wi^{e}_{rms} 1. We note that the values of both Wi^{s}_{rms} and Wi^{e}_{rms} are near or larger than 1, which suggests that the ßow is able to generate suffcient polymer stretching [43,44].

IV. CONCLUSION

In summary, we investigated the ßow of a viscoelastic ßuid in a parallel shear geometry at low Re. This ßow becomes unstable via a nonlinear subcritical instability above a critical Weissenberg number ($Wi_c = 5.2$) if perturbations are strong enough [8]. Using spatially and temporally resolved velocimetry, we identiÞed signatures of elastic turbulence in the parallel shear region. This ßow contrasts in many ways with elastic turbulence near the array of cylinders, which we had found in our previous experiments (same experimental setup) to be linearly unstable [8]. SpeciÞcally, we found that the ßow near cylinders is organized by streamwise streaks that manifest as spots in Fig. 1(b), while temporal bursts that manifest as spanwise bands are found in the parallel shear region [Fig. 1(c)]. Moreover, the energy contained in the high-frequency range near the cylinders seems to shift toward the low-frequency range in the parallel shear region [Fig. 3(a)]. We provided a simple mechanism for the sustained (and even growth) of velocity ßuctuations in the parallel shear region (

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